MATH 211 Fall 2018

TEST II

PART I. Short Answer Questions (One point Each)

a. Write the number $(11011001)_2$ in base 3

b. Write the number $(673)_8$ in hexadecimal.

c. The function $f(n) = (n - n^3 - 3)(log(3n) - 2^n)$ is big Oh of what?

d. The function $g(x) = \dfrac{x^4 + 5}{1 - x + x^2} + (2xlogx - 3x + 7)^2$ is big Oh of what?

e. What numbers must be checked (as divisors or not divisors) to see whether the integer 623 is prime or not?

f. Find the remainder when the integer 345789646 is divided by $2, 3, 6, 7$? (show your reasoning by using divisibility tests)

g. Given $a \equiv 4(\mod 17)$ and $b \equiv 10(\mod 17)$. Then $a^2 - b^3$ is congruent to what modulo 17?

PART II. Provide Your Justification Carefully.

(1) (1.5 points) Write a pseudo-code that takes a list of integers, not necessarily all positive, and returns the number of positive terms that are even (that is, it ignores any negative terms and any odd terms.)

(2) (1.5 point) Suppose that we are using the binary search algorithm to search for the number x in a sorted list of length 50. Suppose further that a_{11} is indeed contains the value of x. Show exactly how the algorithm changes the values of the variables i, j, m etc. to finally locate x. Count the number of comparisons made until this task is completed.

(3) Given is the following algorithm

```
procedure proc(a_1, ··· a_n: positive integers)
ok:=1
i:=2
while(i<=n and ok=1)
    j:=1
    while(a_j ≠ a_i)
        j:= j+1
    if j<i then ok:=0
    else i:= i+1
if ok=0 then return i
else return 0
```

(a) (1.5 pt.) Give the intermediate values of the variables as well as the final result when the input list $4, 2, 0, 2, 1$ is given.

(b) (1.5 pt.) Describe in general what this algorithm returns.

(4) (2 points) Prove that every positive integer n bigger than one is either prime or has a prime divisor that is at most $\sqrt{n}$.

(5) (1.5 point) Show that 2^n is not $\Omega(3^n)$

(6) (2 points) Find a function $g(n)$ such that $1^3 + 2^3 + \ldots + n^3$ is $\Theta(g(n))$ and prove your claim.

(7) (1.5 points) Given is a function $f : A \to B$ and an arbitrary subset T of B. Prove that $f(f^{-1}(T))$ is equal to T when f is a surjective function, (showing exactly where the assumption of surjectivity is needed)

(8) Let S be a non-empty set and consider the function $\mathfrak{F}$ from $\mathcal{P}(S)$ to $\mathcal{P}(S)$ defined as $\mathfrak{F}(A) = S \backslash A$, for any subset A of S.

(a) (1 points) Prove or disprove that $\mathfrak{F}$ is one-to-one.

(b) (1 point) Prove or disprove that $\mathfrak{F}$ is onto.